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Frequency response of constant-current film anemometers

ANTHONY DEMETRIADES
Department of Mechanical Engineering, Montana State University, Bozeman, MT 59715, U.S.A.

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TURBULENCE research in high-speed flows has motivated several new measuring methods [1–3], and has also recruited some older techniques already in use for low-speed flows such as thermoelement anemometers ('hot wires' and 'hot films'). Among the latter, film anemometers are especially attractive because their structural rigidity makes them durable in the hostile surroundings of hypervelocity streams. In such environments, however, film anemometers must also demonstrate the proper response to high-frequency signals. The frequency response of flow-immersible film probes therefore acquires new importance and is discussed in this note.

The fundamentals of this problem were addressed long ago by Ling [4] and Ling and Hubbard [5]. The thermal inertia of the thermoelement alone, without a supporting substrate, causes a signal attenuation of 6 db per frequency octave; Ling [4] showed that with a conducting substrate, the attenuation decreases to 3 db octave^{-1}, without, however, presenting the non-dimensional variables and their range for which this conclusion is valid. Since then, moreover, other workers [6–8] have found that the film-probe response can in fact be a very complex function of the frequency, depending on the thermal boundary conditions and the operating constraints (e.g. 'constant-temperature' or 'constant-current' operation). There exists, therefore, some confusion about the basic rules and parameters controlling the response of film anemometers, including those of simple geometry. In this note we are interested in the generic geometry of the long cylindrical probe, parallel to the flow and with a very small metallic film deposited on its sharpened upstream end. For such probes, which have been successfully operated at constant current at hypersonic speeds [9], the response to high-frequency flow fluctuations is especially critical.

The present analysis, details of which appear in ref. [10], aimed at finding the identity and range of the non-dimensional parameters for which the 3 db octave^{-1} response prevails. In common with most previous approaches, the analysis views the film to be so thin that it practically coincides with the surface of the substrate. The power balance equation for the film includes the ohmic heating, the convective heat exchange with the flow and the conductive exchange with the substrate. Radiation is neglected, and the heat flow in the substrate is one-dimensional. The film–substrate system receives a heat input from the flow which fluctuates with a magnitude much smaller than its corresponding mean level. The fluctuations, which may be due to temperature or Reynolds number changes, are assumed stationary and thus decomposable into Fourier components.

Without thermal lag, the film output as a function of time t would be $e_i \sin \omega t$ due to the Fourier component of frequency ω, where e_i stands for the film temperature, or for some electrical property such as its voltage at constant current. The thermal lag distorts this output into $e_o \sin(\omega t+\phi)$. The present analysis gives the following solutions for e_o and ϕ:

$$\frac{e_o}{e_i} = \frac{1}{[(\omega\tau+Q\sqrt{(\omega\tau)})^2+(1+Q\sqrt{(\omega\tau)})^2]^{1/2}} \quad (1)$$

$$\phi = \tan^{-1}\frac{\tau\omega+Q\sqrt{(\omega\tau)}}{1+Q\sqrt{(\omega\tau)}} \quad (2)$$

which are plotted in Figs. 1 and 2 respectively.

Equations (1) and (2) conveniently separate the effects of the film itself from those of the substrate, upon the attenuation factor e_o/e_i and phase lag ϕ. The film is represented by its 'inherent' time constant τ caused by its finite heat capacity ε (energy per degree). This time constant also depends on the film lateral dimension ('width') w, the fluid thermal conductivity at stagnation conditions k_o, the ratio of 'cold' (unheated or equilibrium) to heated resistance R_c/R, the film Nusselt number N and the logarithmic derivative $N_r = (r/N)(\partial N/\partial r)$ of N relative to $r = (R/R_c - 1)$:

$$\tau = \frac{\varepsilon}{wk_oN\left(\frac{R_c}{R}+N_r\right)}. \quad (3)$$

This expression is, in fact, the classical one for the time constant of any convection-controlled thermoelement such as 'hot wire' anemometers [11].

The effect of the substrate enters via the non-dimensional 'loss factor' Q, which depends on the substrate conductivity, density and specific heat, k_s, ρ_s, and c_s respectively:

$$Q = \left[\frac{1}{2N}\frac{k_s}{k_o}\frac{\rho_s c_s}{\rho_f c_f}\frac{h}{d}\right]^{1/2}. \quad (4)$$

The film is also represented in equation (4) by its dimensions (height h and depth or thickness d) and material density ρ_f and specific heat c_f. Note that the theory admits a finite mass

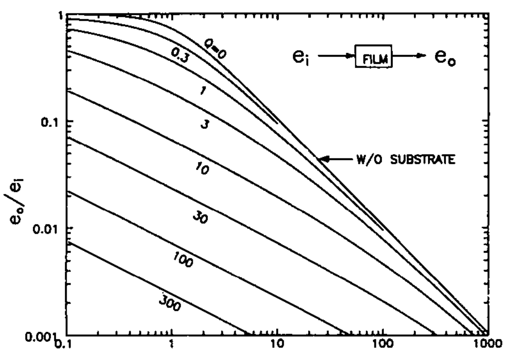

FIG. 1. Signal attenuation as a function of the non-dimensional frequency and the substrate loss factor Q.

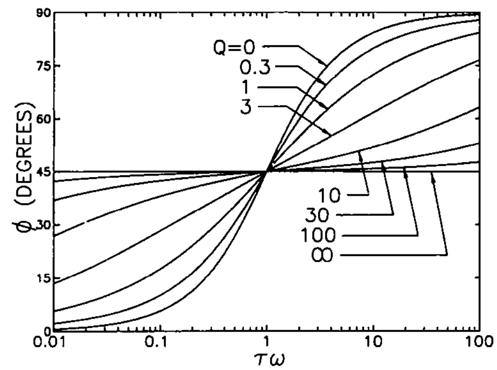

FIG. 2. Phase lag as a function of the non-dimensional frequency and the substrate loss factor Q.

for the film but a thickness d which is vanishingly small compared to all other physical dimensions.

According to Fig. 1, the film response is controlled by the inherent time constant alone for small Q; the curve $Q = 0$ is the classic 'hot wire' response of the early anemometer literature [11]. At the other extreme of large Q the attenuation is already substantial at low frequencies, behaves as 3 db octave^{-1} at intermediate to high frequencies and returns to 6 db octave^{-1} at very large $\omega\tau$ (this asymptotic behavior simply indicates that the 'thermal skin depth' becomes so small that the substrate also becomes unimportant). In such cases where $Q \gg 1$ and $Q \gg \sqrt{(\omega\tau)}$ at the same time, equation (1) becomes $e_o/e_i = 1/Q\sqrt{(2\omega\tau)}$, in contrast to the $1/\omega\tau$ limit for $Q = 0$.

The inherent time constant can be made very small in actual film anemometers, but this advantage is more than offset by the loss factors encountered for typical materials. Probes fabricated by this author with 0.004-cm-wide, 0.01-cm-long platinum films had an inherent time constant of 2.1 μs when operated at low 'overheats' ($r \approx 0$) and at a Nusselt number N of 30 and stagnation flow temperature of 273 K. These films were deposited on a quartz substrate, for which the loss factor Q at the same conditions were computed at 138. For these values of τ and Q, equation (1) gives a ratio e_o/e_i of 0.044 at a frequency of 1 kHz, and a ratio of 0.0014 at 1 MHz. As with hot wire anemometers these attenuation levels are usually unacceptable and require thermal-lag compensation by electronic means. Film probes, therefore, offer little in the way of improving the frequency response, although their ruggedness and durability make them attractive for measurements in flows with high pressures and temperatures.

In summary, equations are presented here for the frequency response and phase lag of a constant-current, stagnation-point film anemometer. By conveniently isolating the two important parameters which control this response, it is possible to understand the probe behavior in different frequency regimes, and to design the film and substrate consistently with the time resolution requirements of the flow field. Finally, a prediction of the response allows the design of electronic circuits suitable for thermal-lag compensation, as was done in the early stages of hot wire anemometry [11].

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